Aston University

School of Engineering and Applied Science

SE0EM1 – Mathematics 1

Foundation Year Class Test

CLOSED BOOK

Date: ** 2017
Time: **
Duration: 2 hours

Instructions to Candidates

1. Answer ALL questions in Section A
2. Answer two questions from Section B
3. The marks for each question are listed on the right of each question
4. Use of calculators is allowed

Materials provided

1. Answer booklet
2. Formula sheet is attached this question paper

This exam paper must not be removed from the exam room

Section A: Answer all questions

A1. a. Determine the Highest Common Factor and Lowest Common Multiple of the following set of numbers

7350, 6860, 2700 — 4 marks

b. Use a valid technique to show your answers are correct. — 2 marks

A2. Express the following in terms of powers of prime numbers:

a. $\dfrac{4^{-3} \times 4^{4}}{4^{3} \times 4^{6}}$ — 3 marks

b. $\dfrac{2^{3} \times 6^{6}}{4^{-3} \times 5^{2}}$ — 3 marks

A3. Show the following numbers in binary, decimal and hexadecimal:

$1111\ 1100_b$, 215_d, $C4_h$ — 6 marks

A4. Expand the following expressions giving your answer in decreasing powers of x.

a. $(2x - 4)^2(x + 6)$ — 3 marks

b. $(5x + 3)^3$ — 3 marks

A5. a. Carry out the following algebraic division.

$$\frac{20x^3 + 47x^2 + 36x + 9}{4x + 3}$$

4 marks

b. Prove that your answer is correct. — 2 marks

A6. a. Factorise the following cubic polynomial

$18x^3 + 66x^2 + 38x + 6$ 4 marks

b. Show that you answer is correct 2 marks

A7. a. Solve the simultaneous equations to find the points of intersection.

$y = -x^2 + 9x + 15$

$y = 5x + 3$ 4.marks

b. Prove that your answer is correct. 2 marks

A8. a. Separate the following into partial fractions

$\frac{-4x - 18}{-5x^2 + 23x - 12}$ 4 marks

b. In the simplest manner prove your answer is correct. 2 marks

A9. a. Transform the expression $6\sin\theta + 12\cos\theta$ to the form $r\sin(\theta + \phi)$ giving ϕ in the range 0 to π. 4 marks

b. Sketch $r\sin(\theta + \phi)$. 2 marks

A10. Given $A = \begin{bmatrix} -3 & 2 \\ 7 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 4 \\ 2 & -3 \end{bmatrix}$ and $C = \begin{bmatrix} 7 \\ -6 \end{bmatrix}$ find the following:

a. $A - B$ 1 marks

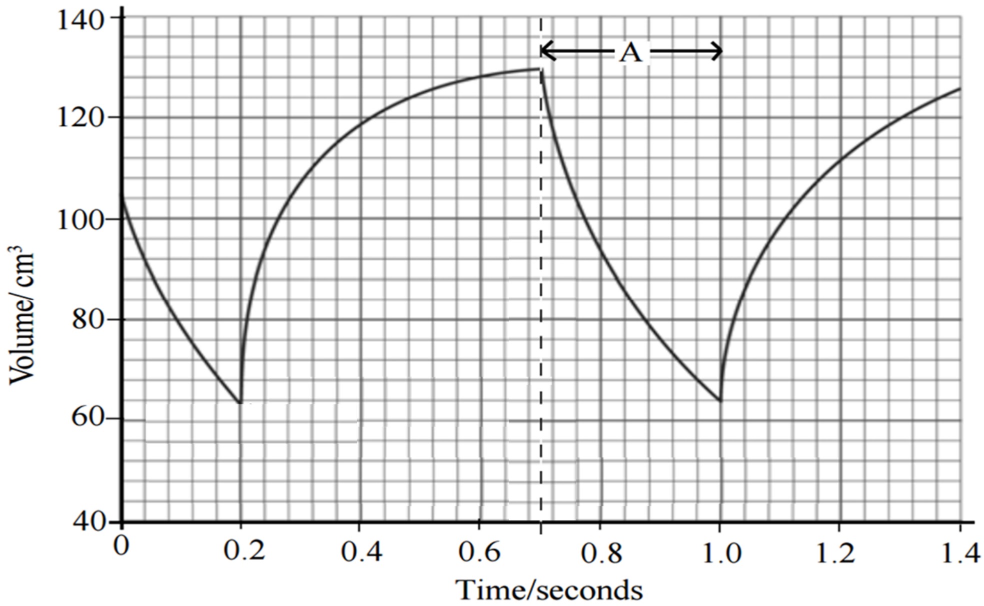
b. $|C|$ 1 marks

c. $|B|$ 1 marks

d. $B \times A$ 3 marks

Section B: Answer 2 questions

B1. The following graph show changes in the volume of blood in the right ventricle as the heart beats.



- Stroke volume (SV) is the volume of blood pumped from the ventricle per beat.
- Heart Rate (HR) is the number of beats per minute.
- Cardiac Output = Stroke Volume × Heart Rate

a. The line A shows when blood is leaving the ventricle. Explain, in terms of blood pressure, why blood does not flow back into the atrium during this part of the cycle.

4 marks

b. Draw a line on the graph to show one complete cardiac cycle.

4 marks

c. Draw a line on the graph to show the period in one cardiac cycle when the muscle in the wall of the ventricle is relaxed.

4 marks

d. Using your answer to part b. calculate the number of times the heart would beat in one minute. Show your workings.

4 marks

e. Calculate the volume of blood pumped out by the heart in one minute (cardiac output) in dm^3. Show your workings.

4 marks

B2. a. Explain, with examples what is meant by **significant figures**. 4 marks

b. Give the number of significant figures in each of the following

i. 3.08000 2 marks

ii. 0.00418 2 marks

iii. 7.09×10^{-5} 2 marks

c. The following data were collected from experiments. Calculate the final value giving your answer to the correct number of significant figures.

$$\frac{561.0 \times 34{,}908 \times 23.0}{21.888 \times 75.2 \times 120.00}$$

10 marks

B3. a. Find the units of the molar gas constant R using the ideal gas equation

$$pV = nRT$$

given that p is measured in kg m^{-1} s^{-2}, V in m^3, n in mol and T in K.

10 marks

b. The Arrhenius equation, given below, describes the relationship between the rate of a reaction k and the temperature T

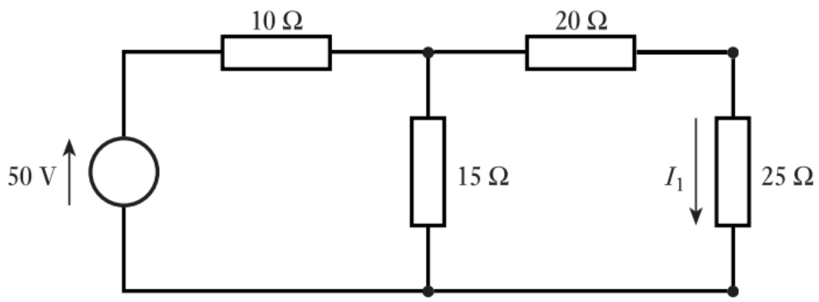
$$k = Ae^{\left(\frac{-E_a}{RT}\right)}$$

where E_a, R and A are all constants.

Given the activation energy $E_a = 52.0\ kJ\ mol^{-1}$, the gas constant $R = 8.31\ JK^{-1}mol^{-1}$ and $A = 1.00$ what is the rate of the reaction k when the temperature $T = 241\ K$?

10 marks

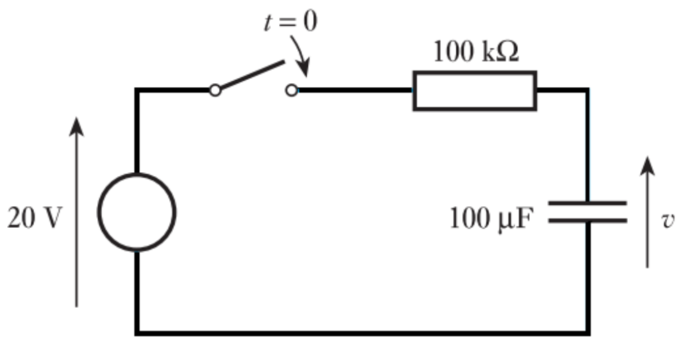
B4. a. Given that for resistors in series $R_T = R_1 + R_2$ and for resistors in parallel $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$ calculate the current I_1 in the following circuit.



10 marks

b. The switch in the following circuit is closed at $t = 0$. The voltage across the capacitor V_c is given by $V_c = V(1 - e^{-t/RC})$ where t is time, R is the resistance and C is the capacitance.

i. Calculate the voltage on the capacitor at $t = 25$ s.
ii. How can you increase the rate at which the capacitor charges?



10 marks

B5. a. A prescription specifies the dose to be 2g of a specific mixture. Your stock of the mixture is labelled 500mg in 10ml where 500mg is the stock strength and 10ml is the stock volume.

How much mixture should you administer?

10 marks

b. A dehydrated person is prescribed 1.5L of rehydration fluid over a 10 hour period? Your equipment delivers 20 drops per mL.

What drip rate in drops per minute will supply the prescription?

10 marks

B6. a. Solve the following simultaneous equations using Cramer's method

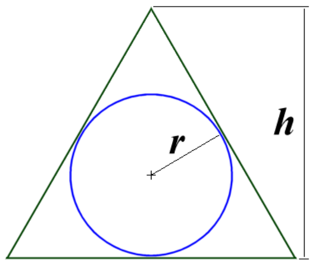
$$2x + 4y + 6z = 22$$
$$3x - 3y + 4z = 4$$
$$2x + 5y + 5z = 19$$

12 marks

b. Show that your answers are correct. 8 marks

B7. a. An equilateral, triangular frame is made to fit around a ball of radius r.

Show that the height h of the frame is equal to $3r$.



6 marks

b. How would you confirm the answer is correct? 2 marks

B8. a. The manufacturer of a board game printed $18,500,000,000,000 of toy money in 1990. If this this toy money were distributed equally among the world's population, estimated to be 5.3 billion in 1990, how much would each person get?

8 marks

b. If the toy money was distributed so that the poorest person got the most, the richest person got nothing and there is a linear variation between the two how much would the person in the middle get?

12 marks

END OF QUESTIONS

END OF PAPER

Algebra

$(x+k)(x-k) = x^2 - k^2$

$(x+k)^2 = x^2 + 2kx + k^2$

Formula for solving quadratic equations

If $ax^2 + bx + c = 0$ then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Laws of Indices

$a^m . a^n = a^{m+n}$	$\frac{a^m}{a^n} = a^{m-n}$
$(a^m)^n = a^{mn}$	$a^0 = 1$
$a^{-m} = \frac{1}{a^m}$	$a^{1/m} = \sqrt{a^m}$

Laws of Logarithms

For any base b where b > 1

$log_b A = c$ means $A = b^c$

$log_b A + log_b B = log_b AB$

$log_b A - log_b B = log_b \frac{A}{B}$

$n log_b A = log_b A^n$

$log_b 1 = 0$

$log_b b = 1$

Partial Fractions

For proper fractions $\frac{P(x)}{Q(x)}$ where P and Q are polynomialswith the degree of P less than the degree of Q:

a linear factor $ax + b$ in the denominator produces a partial factor of the form $\frac{A}{ax+b}$

a repeated linear factor $(ax+b)^2$ in the denominator produces a partial factor of the form $\frac{A}{ax+b} + \frac{B}{(ax+b)^2}$

a quadratic factor $ax^2 + bx + c$ in the denominator produces a partial factor of the form $\frac{Ax+B}{ax^2+bx+c}$

Improper fractions require an additional term which is a polynomial of degree $n - d$ where n is the degree of the numerator and d is the degree of the denominator.

Trigonometry

360° = 2π radians

Radians to degrees: $\times 180 \div \pi$

Degrees to radians: $\times \pi \div 180$

Trigonometric ratios for an acute angle θ

$$sin\theta = \frac{opposite}{hypotenuse}$$

$$cos\theta = \frac{adjacent}{hypotenuse}$$

$$tan\theta = \frac{opposite}{adjacent}$$

Common Trigonometric Identities

$\sin(\theta \pm \phi) = sin\theta . cos\phi \pm cos\theta . sin\phi$

so $\sin 2\theta = 2 sin\theta . cos\theta$

$\cos(\theta \pm \phi) = cos\theta . cos\phi \mp sin\theta . sin\phi$

so $\cos 2\theta = cos^2\theta - sin^2\theta$

$$\tan(\theta \pm \phi) = \frac{tan\theta \pm tan\phi}{1 \mp tan\theta . tan\phi}$$

$2 \sin\theta . cos\phi = \sin(\theta - \phi) + \sin(\theta + \phi)$

$2 \cos\theta . cos\phi = \cos(\theta - \phi) + cos(\theta + \phi)$

$2 \sin\theta . sin\phi = \cos(\theta - \phi) - \cos(\theta + \phi)$

$$sin^2\theta = \frac{1 - cos2\theta}{2}$$

$$cos^2\theta = \frac{1 + cos2\theta}{2}$$

$cos^2\theta + sin^2\theta = 1$

$cos^2\theta - sin^2\theta = \cos 2\theta$

$tan^2\theta + 1 = sec^2\theta$

$1 + cot^2\theta = cosec^2\theta$